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Book Review: *Theory of Critical Phenomena in Finite-Size Systems, Scaling and Quantum Effects*

Theory of Critical Phenomena in Finite-Size Systems, Scaling and Quantum Effects. J. G. Brankov, D. M. Danchev, and N. S. Tonchev, Series in Condensed Matter Physics, Vol. 9, World Scientific, 2000.

This book is the ninth volume in World Scientific's Series in Modern Condensed Matter Physics Series which includes contributions by, e.g., P. G. de Gennes. It is fundamentally devoted to a single question: What happens to many-body systems when the range of correlations is on the order of the system size ?

Because of spectacular increases in our basic understanding of the topic, especially since the 1970's, that question is currently most likely to provoke a discussion on finite-size effects in phase transitions. The appearance of Brankov, Dancehev, and Tonchev's (BDT) book signals that research on this topic has matured sufficiently that textbooks and monographs are now appropriate.

Paying little attention to experimental verification and focusing on sophisticated (spin) model analysis, BDT stress the importance of finite-size scaling near bulk phase transitions and describe how boundary and Casimir effects appear in confined systems near criticality. The discussion centers on classical model systems, but simple models of ($T = 0$) "quantum-critical" behavior are studied too.

Due to this narrow choice of topics, and due to an ambitious theoretical approach, this book cannot be expected to have a broad readership. Students and non-experts will find the presententation too technical. More importantly, perhaps, the book will probably fail to excite physicists with interests in soft condensed matter and physical chemists with research interests in, e.g., correlated fluids, polymers and liquid crystals, for whom the interplay between long-range correlations and system size is always an integral part of the research.

Some expert readers with a background in condensed matter and statistical physics may find BDT's choice of topics and style of presentation to

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be on the conservative side, yet I imagine that the book will be popular among experts with a strong background in critical phenomena who fancy careful model analysis as a means of examining and testing major theoretical ideas. Those experts will hopefully agree with me that BDT's testing of many different aspects of phenomenological finite-size scaling theory by confronting its predictions with exact results from the spherical model is carried out in a very coherent and satisfactory way.

The organization of the book is as follows: Roughly one third of the material (Chapters 1–3, 10) deals with bulk phase transitions and critical phenomena, classical as well as ($T = 0$) quantum-mechanical: In Chapter 1, BDT introduce us to most of the ingredients in the modern discussion of phase transitions and critical phenomena, including the hypothesis of (hyper)scaling and universality, and the renormalization group language. The presentation is very readable. Chapters 2 and 3 are devoted to the business of "model building": A rather formal and involved presentation is given of Bogolyubov's method of approximating Hamiltonians, followed by a more satisfactory description of some exactly solved spin models. As the reader soon realizes, the spherical model plays a special role here as it does in later chapters while, interestingly, no mention is made of the Ising model.

Bulk "quantum-critical" phenomena are described in Chapters 3 and 10. The main idea is not so much to develop a comprehensive theory of "quantum-critical" behavior, controlled in a narrow temperature range near $T = 0$ by quantum rather than thermal fluctuations. Instead the authors wish to take advantage of an (obvious) exact relation between finite-size scaling theory of classical $d+1$ dimensional critical slabs of width $\hbar/k_B T$ and d-dimensional "quantum-critical" systems. A careful study of quantum versions of the ϕ^4- and spherical-models illustrate the usefulness of the approach.

Another one third of the book (Chapters 4–6 and 8–9) is devoted to finite-size scaling near bulk phase transitions. This is probably the most important part of the book. Significant attention is paid to finite-size scaling at criticality below (Chapter 4) and above (Chapter 6) the upper critical dimension, where the appearence of "dangerous irrelevant variables" invalidates the hyperscaling hypothesis as well as basic elements of conventional finite-size scaling theory. Finite-size scaling at first order phase transitions is described in Chapter 8. The presentation well-organized: In each chapter, relevant aspects of phenomenological finite-size scaling theory are explained in great detail; then theory is tested against exact results from the spherical model. The presentation is also very convincing: One gets the sense that most relevant questions about the "rounding" and shifts of phase transitions in finite systems have been addressed. Still, for the sake of completeness, it would have been nice, had the authors told us that we know that finite-size scaling theory works, not only because it agrees with exactly solved models, but also because it agrees with, e.g., numerous computer simulation studies, and, in $d = 2$ dimensional space, with predictions from conformal field theory.

The remaining third of the book is concerned with boundary and interface effects (Chapter 7) and with the forces between interfaces due to confinement of critical fluctuations, i. e., thermal Casimir forces (Chapters 11–12). The study of boundary effects focuses on the degree to which the finite-size scaling hypothesis applies to critical slabs of thickness L. Unsurprisingly, a model study involving the spherical model confirms that it does. The chapters on the thermal Casimir effect are more ambitious: In Chapter 11, analysis of classical and quantum spherical models helps us better understand the nature of the thermal Casimir effect. In Chapter 12, various other theoretical results on the Casimir effect are surveyed so that a qualified discussion of the experimental situation can be carried through.

In my view, BDT manage to make the discussion of the Casimir effect interesting, if sometimes too focused on Casimir amplitudes. The discussion of boundary effects, on the other hand, is disappointing: Despite the fact that the material presented in Chapter 7 is indeed relevant, one gets the feeling that the question of how critical behavior is modified due to the presence of boundaries and surface fields is answered incompletely. For instance, it is hard for me to see how the authors can ignore the highly relevant problem, which has been studied by Cahn, Fisher, Lipowsky and others, in which one of the interfaces in a near-critical slab is allowed to fluctuate and to *wander* so that the slab thickness can grow (or shrink). More generally, wetting and layering near a critical point is completely ignored by BDT. I fail to understand why.

In summary, this book offers a careful survey of finite-size scaling near bulk phase transitions, but fails when it comes to providing a fully satisfactory overview of boundary effects in systems where long-range correlations and critical-point fluctuations are important.

The implication is, as I have mentioned above, that the book is likely to satisfy a group of expert readers whose interests center on bulk phase transition, but unlikely to satisfy the curiousity of those soft condensed matter physicists and physical chemists who understand the importance of the topics discussed in this book, but find that their interests center on boundary effects.

One hopes that future books concerned with boundary effects and with the influence of size and confinement in systems with long-range correlations, will seek a better balance in presentation of the various effects; more adventurous authors might also find that subjects other than bulk critical phenomena, e.g., the science of polymers and liquid crystals, can inspire the discussion of finite-size effects in substantial ways.

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